

Sixth Grade Mathematics

By the end of grade six, students have mastered the four arithmetic operations with whole numbers, positive rational numbers, positive decimals, and positive and negative integers; they accurately compute and solve problems. They find prime factorizations, least common multiples, and greatest common factors. They create, evaluate, and simplify expressions, and solve equations involving two operations and a single variable. They solve problems involving an unknown angle in a triangle or quadrilateral, and use properties of complementary and supplementary angles. Students know about π as the ratio between the circumference and the diameter of a circle and solve problems using the formulas for the circumference and area of a circle. Students analyze, draw conclusions, and make predictions based upon data and apply basic concepts of probability.

Curriculum Focal Points

- *Number and Operations*: Developing an understanding of and fluency with multiplication and division of fractions and decimals.
- *Number and Operations*: Connecting ration and rate to multiplication and division.
- *Algebra*: Writing, interpreting, and using mathematical expressions and equations.
(See Appendix I for complete document)

Standard 1: Students will expand number sense to include operations with rational numbers.

Objective 1: Represent rational numbers in a variety of ways.

- Recognize a rational number as a ratio of two integers, a to b , where b is not equal to zero.
- Change whole numbers with exponents to standard form (e.g., $24 = 16$) and recognize that any non-zero whole number to the zero power equals 1 (e.g., $90 = 1$).
- Write a whole number in expanded form using exponents (e.g., $876,539 = 8 \times 10^5 + 7 \times 10^4 + 6 \times 10^3 + 5 \times 10^2 + 3 \times 10^1 + 9 \times 10^0$).
- Express numbers in scientific notation using positive powers of ten.

Objective 2: Explain relationships and equivalencies among rational numbers.

- Place rational numbers on the number line.
- Compare and order rational numbers, including positive and negative mixed fractions and decimals, using a variety of methods and symbols, including the number line and finding common denominators.
- Find equivalent forms for common fractions, decimals, percents, and ratios, including repeating or terminating decimals.
- Relate percents less than 1% or greater than 100% to equivalent fractions, decimals, whole numbers, and mixed numbers.
- Recognize that the sum of an integer and its additive inverse is zero.

Objective 3: Use number theory concepts to find prime factorizations, least common multiples, and greatest common factors.

- Determine whether whole numbers to 100 are prime, composite, or neither.

- b. Find the prime factorization of composite numbers to 100.
- c. Find the greatest common factor and least common multiple for two numbers using a variety of methods (e.g., list of multiples, prime factorization).

Objective 4: Model and illustrate meanings of operations and describe how they relate.

- a. Relate fractions to multiplication and division and use this relationship to explain procedures for multiplying and dividing fractions.
- b. Recognize that ratios derive from pairs of rows in the multiplication table and connect with equivalent fractions.
- c. Give mixed number and decimal solutions to division problems with whole numbers.

Objective 5: Solve problems involving multiple steps.

- a. Select appropriate methods to solve a multi-step problem involving multiplication and division of fractions and decimals.
- b. Use estimation to determine whether results obtained using a calculator are reasonable.
- c. Use estimation or calculation to compute results, depending on the context and numbers involved in the problem.
- d. Solve problems involving ratios and proportions.

Objective 6: Demonstrate proficiency with the four operations, with positive rational numbers, and with addition and subtraction of integers.

- a. Multiply and divide a multi-digit number by a two-digit number, including decimals.
- b. Add, subtract, multiply, and divide fractions and mixed numbers.
- c. Add and subtract integers.

Mathematical Language and Symbols Students Should Use

prime, composite, exponent, least common multiple, least common denominator, greatest common factor, decimals, percents, divisible, divisibility, equivalent fractions, integer, dividend, quotient, divisor, factor, simplest terms, mixed numeral, improper fraction

Exploratory Concepts and Skills

- √ Explore the addition and subtraction of positive and negative fractions.
- √ Investigate the concepts of ratio and proportion.
- √ Investigate the distributive property of multiplication over addition of double-digit multipliers.

Standard 2: Students will use patterns, relations, and algebraic expressions to represent and analyze mathematical problems and number relationships.

Objective 1: Analyze algebraic expressions, tables, and graphs to determine patterns, relations, and rules.

- a. Describe simple relationships by creating and analyzing tables, equations, and expressions.
- b. Draw a graph and write an equation from a table of values.
- c. Draw a graph and create a table of values from an equation.

Objective 2: Write, interpret, and use mathematical expressions, equations, and formulas to represent and solve problems that correspond to given situations.

- a. Solve single variable linear equations using a variety of strategies.
- b. Recognize that expressions in different forms can be equivalent and rewrite an expression to represent a quantity in a different way.
- c. Evaluate and simplify expressions and formulas, substituting given values for the variables (e.g., $2x + 4$; $x = 2$; therefore, $2(2) + 4 = 8$).

Mathematical Language and Symbols Students Should Use

Order of operations, sequence, function, pattern, algebraic expression, approximately equal, \approx , notation for exponents: 4^3 or 4^3 , a number in front of a variable indicates multiplication (e.g., $3y$ means 3 times the quantity y), formula, generalization

Exploratory Concepts and Skills

- ✓ Use physical models to investigate and describe how a change in one variable affects a second variable.
- ✓ Use models to develop understanding of slope as constant rate of change.
- ✓ Model situations with proportional relationships and solve problems.

Standard 3: Students will use spatial and logical reasoning to recognize, describe, and analyze geometric shapes and principles.

Objective 1: Identify and analyze attributes and properties of geometric shapes to solve problems.

- a. Identify the midpoint of a line segment and the center and circumference of a circle.
- b. Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.
- c. Develop and use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle in a triangle or quadrilateral.

Objective 2: Visualize and identify geometric shapes after applying transformations on a coordinate plane.

- a. Rotate a polygon about the origin by a multiple of 90° and identify the location of the new vertices.
- b. Translate a polygon either horizontally or vertically on a coordinate grid and identify the location of the new vertices.
- c. Reflect a polygon across either the x - or y -axis and identify the location of the new vertices.

Mathematical Language and Symbols Students Should Use

midpoint, circumference, complementary and supplementary angles, rotate, translate, reflect, transformation.

Exploratory Concepts and Skills

- √ Use manipulatives and technology to model geometric shapes.
- √ Investigate tessellations.
- √ Explore the angles formed by intersecting lines.
- √ Identify and draw shapes and figures from different views/perspectives.

Standard 4: Students will understand and apply measurement tools and techniques and find the circumference and area of a circle.

Objective 1: Describe and find the circumference and area of a circle.

- a. Explore the relationship between the radius and diameter of a circle to the circle's circumference to develop the formula for circumference.
- b. Find the circumference of a circle using a formula.
- c. Describe pi as the ratio of the circumference to the diameter of a circle.
- d. Decompose a circle into a number of wedges and rearrange the wedges into a shape that approximates a parallelogram to develop the formula for the area of a circle.
- e. Find the area of a circle using a formula.

Objective 2: Identify and describe measurable attributes of objects and units of measurement, and solve problems involving measurement.

- a. Recognize that measurements are approximations and describe how the size of the unit used in measuring affects the precision.
- b. Convert units of measurement within the metric system and convert units of measurement within the customary system.
- c. Compare a meter to a yard, a liter to a quart, and a kilometer to a mile.
- d. Determine when it is appropriate to estimate or use precise measurement when solving problems.
- e. Derive and use the formula to determine the surface area and volume of a cylinder.

Mathematical Language and Symbols Students Should Use

cylinder, radius, diameter, circumference, area, surface area, volume, π

Exploratory Concepts and Skills

- √ Investigate volumes and surface areas of a variety of three-dimensional objects.

Standard 5: Students will analyze, draw conclusions, and make predictions based upon data and apply basic concepts of probability.

Objective 1: Design investigations to reach conclusions using statistical methods to make inferences based on data.

- a. Design investigations to answer questions.
- b. Extend data display and comparisons to include scatter plots and circle graphs.
- c. Compare two similar sets of data on the same graph and compare two graphs representing the same set of data.
- d. Recognize that changing the scale influences the appearance of a display of data.

- e. Propose and justify inferences and predictions based on data.

Objective 2: Apply basic concepts of probability and justify outcomes.

- a. Write the results of a probability experiment as a fraction between zero and one, or an equivalent percent.
- b. Compare experimental results with theoretical results (e.g., experimental: 7 out of 10 tails; whereas, theoretical 5 out of 10 tails).
- c. Compare individual, small group, and large group results of a probability experiment in order to more accurately estimate the actual probabilities

Mathematical Language and Symbols Students Should Use

data display, scatter plot, circle graph, scale, predict, justify, probability, experimental results theoretical results

Math 7

Students in Math 7 will study mathematics concepts from sixth grade in more depth and extend knowledge to basic pre-algebra by conjecturing, verifying, thinking critically, and applying mathematical concepts. This course focuses on computation and estimation with rational numbers and emphasizes proportional reasoning. Students will investigate and explore mathematical ideas using technology and models to develop multiple strategies for analyzing complex situations. Students will apply mathematical skills and make meaningful connections to life's experiences.

Curricular Focal Points

- Number and Operations and Algebra and Geometry: Developing an understanding of and applying proportionality, including similarity.
- Measurement and Geometry and Algebra: Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes.
- Number and Operations and Algebra: Developing an understanding of operations on all rational numbers and solving linear equations.
(See Appendix I for complete document)

Standard 1: Students will expand number sense to understand, perform operations, and solve problems with rational numbers.

Objective 1: Represent rational numbers in a variety of ways.

- a. Demonstrate multiple ways to represent whole numbers, decimals, fractions, percents, and integers using models and real-life examples.
- b. Simplify numerical expressions with whole number exponents using order of operations, and recognize that any positive number to the 0 power is 1.
- c. Represent numbers greater than one using scientific notation.
- d. Select the most appropriate form of a rational number for a given context.

Objective 2: Compare and order rational numbers, including positive and negative fractions, positive and negative mixed numbers, and positive and negative decimals.

- a. Identify, read, and locate rational numbers on a number line.
- b. Compare pairs of rational numbers in different forms.
- c. Order rational numbers with and without a number line.

Objective 3: Explain relationships and equivalences among rational numbers.

- a. Find equivalent forms for common fractions, decimals, percents, and ratios, including repeating or terminating decimals.
- b. Predict the effect of operating with fractions, decimals, percents, and integers as an increase or a decrease of the original value.
- c. Recognize and use the identity properties of addition and multiplication, the multiplicative property of zero, the commutative and associative properties of addition and multiplication, and the distributive property of multiplication over addition.
- d. Recognize and use the inverse operations of adding and subtracting a fixed number, multiplying and dividing by a fixed number, and computing squares of whole numbers and taking square roots of perfect squares.

Objective 4: Model meanings of ratios and operations with rational numbers.

- a. Demonstrate that the fraction b/a represents a divided by b .
- b. Recognize percents as ratios based on 100 and decimals as ratios based on powers of 10.
- c. Extend the multiplication of whole numbers to multiplication of fractions using area models, measurement models, and the number line.
- d. Compare the division of whole numbers to the division of fractions using area or set models, the number line, and missing factors.

Objective 5: Solve problems involving rational numbers.

- a. Compute fluently using all four operations with integers and positive fractions and decimals.
- b. Solve problems using factors, multiples, prime factorization, relatively prime numbers, and common divisibility rules.
- c. Solve application problems involving rational numbers.
- d. Determine if an answer is reasonable using estimation.

Mathematical Language and Symbols Students Should Use

whole number, decimal, fraction, percent, integer, exponent, scientific notation, rational number, identity, commutative, associative, distributive, factor, multiple, prime, relatively prime, additive inverse, multiplicative inverse

Standard 2: Students will use proportional reasoning to solve problems.

Objective 1: Solve problems involving ratios, rates, proportions and percentages.

- a. Solve ratio and rate problems using informal methods involving multiplication and division.
- b. Solve percent problems using ratio and proportion, including problems involving discounts, interest, taxes, tips, and percent increase or decrease.
- c. Solve problems involving proportions, rates, and measures.

- Objective 2: Apply the properties of proportionality to different units of measurement.
- Convert from one unit of measurement to an equivalent unit of measurement in the same system using a given conversion factor.
 - Understand that in a proportional relationship, all dimensions change by the same scale factor.
 - Create and interpret scale drawings and approximate distance on maps using proportions.

Mathematical Language and Symbols Students Should Use

ratio, rate, proportion, scale drawing, conversion factor

Standard 3: Students will develop fluency with the language and operations of Algebra to analyze and represent relationships.

- Objective 1: Evaluate, simplify, and solve algebraic expressions and equations.
- Write a variable expression to identify pattern relationships, and use those expressions to make predictions.
 - Translate verbal expressions into algebraic expressions.
 - Simplify and evaluate algebraic expressions.
 - Show that performing the same operation on both sides of an equation will produce an equivalent equation.
 - Solve single-variable linear equations and inequalities of the form $ax + b = c$, $ax + b < c$, or $ax + b > c$.

- Objective 2: Represent relationships using graphs, tables, and other models.
- Identify integer coordinates when given the graph of a point on a rectangular coordinate system.
 - Graph ordered pairs of integers on a rectangular coordinate system.
 - Model real-world problems using graphs, tables, equations, manipulatives, and pictures.

Mathematical Language and Symbols Students Should Use

variable expression, algebraic expression, equivalent, linear equation, linear inequality, rectangular coordinate system, ordered pair

Standard 4: Students will use algebraic, spatial, and logical reasoning to solve Geometry and measurement problems.

- Objective 1: Draw, label, and describe attributes of geometric figures to determine geometric relationships.
- Draw, label, and describe relationships among line segments, rays, lines, parallel lines, and perpendicular lines, including midpoint of a line segment.
 - Draw, label, and describe relationships among vertical, adjacent, complementary, and supplementary angles.

- c. Draw, label, and describe attributes of angles, triangles, and quadrilaterals.

Objective 2: Determine measurements in metric and customary units using appropriate tools and formulas.

- a. Estimate metric and customary measures using everyday objects and comparisons.
- b. Measure length, area, volume, and angles to appropriate levels of precision.
- c. Calculate the measurement of everyday objects using formulas for perimeters and
and
areas of triangles and quadrilaterals, and circumferences and areas of circles.
- d. Calculate the measurement of everyday objects using formulas for surface area and volume of right triangular and rectangular prisms and cylinders.

Mathematical Language and Symbols Students Should Use

line segment, ray, line, parallel, perpendicular, midpoint, vertical angles, adjacent angles, complementary angles, supplementary angles

Standard 5: Students will understand concepts from probability and statistics and apply statistical methods to solve problems.

Objective 1: Use basic concepts of probability to determine the likelihood of an event and compare the results of various experiments.

- a. Write the results of a probability experiment as a fraction, ratio, or decimal, between zero and one, or as a percent between zero and one hundred, inclusive.
- b. Compare experimental results with theoretical probability.
- c. Compare individual, small group, and large group results of a probability experiment.

Objective 2: Display and compare data to make predictions and formulate conclusions.

- a. Display data using tables, scatter plots, and circle graphs.
- b. Compare two similar sets of data on the same graph.
- c. Compare two different kinds of graphs representing the same set of data.
- d. Propose and justify inferences and predictions based on data.

Mathematical Language and Symbols Students Should Use

experimental result, theoretical probability, scatter plot, circle graph, inference