Third Grade Mathematics

By the end of grade three, students develop understandings of multiplication and division of whole numbers. They use properties to develop increasingly more sophisticated strategies to solve problems involving basic multiplication and division facts. They relate division to multiplication. Students understand fraction equivalence for simple fractions; they recognize that the size of a fractional part is relative to the size of the whole. They understand meanings of fractions to represent parts of a whole, parts of a set, or distances on a number line. They compare and order simple fractions by using models, benchmark fractions, or common denominators.

Students investigate, analyze, and classify two-dimensional shapes by their sides and angles. They decompose, combine, and transform polygons to understand properties of two-dimensional space and use those properties to solve problems. Students construct and analyze frequency tables, bar graphs, picture graphs, and line plots and use them to solve problems.

Curriculum Focal Points
- Number and Operations and Algebra: Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts.
- Number and Operations: Developing an understanding of fractions and fraction equivalence.
- Geometry: Describing and analyzing properties of two-dimensional shapes. (See Appendix I for complete document)

Standard 1: Students will understand the base-ten numeration system, place value concepts, simple fractions and perform operations with whole numbers.

Objective 1: Represent whole numbers up to 10,000, comprehend place value concepts, and identify relationships among whole numbers using base-ten models and symbolic notation.
      a. Read, write, and represent whole numbers using standard and expanded form and word form.
      b. Demonstrate multiple ways to represent numbers using models and symbolic representations (e.g., fifty is the same as two groups of 25, the number of pennies in five dimes, or 75 - 25).
      c. Identify the place and the value of a given digit in a five-digit numeral and round numbers to the nearest ten, hundred, and thousand.
      d. Order and compare whole numbers on a number line and use the symbols <, >, ≠, and = when comparing whole numbers.
      e. Identify factors and multiples of whole numbers.

Objective 2: Use fractions to describe and compare parts of the whole.
      a. Identify the denominator of a fraction as the number of equal parts of the unit whole and the numerator of a fraction as the number of equal parts being considered.
      b. Define regions and sets of objects as a whole and divide the whole into equal parts using a variety of objects, models, and illustrations.
      c. Name and write a fraction to represent a portion of a unit whole for halves, thirds, fourths, sixths, and eighths.

d. Place fractions on the number line and compare and order fractions using models, pictures, the number line, and symbols.

e. Find equivalent fractions using concrete and pictorial representations.

Objective 3: Model problems involving addition, subtraction, multiplication, and division.

a. Demonstrate the meaning of multiplication and division of whole numbers through the use of a variety of representations (e.g., equal-sized groups, arrays, area models, and equal jumps on a number line for multiplication, partitioning and sharing for division).

b. Use a variety of strategies and tools, such as repeated addition or subtraction, equal jumps on the number line, and counters arranged in arrays to model multiplication and division problems.

c. Demonstrate, using objects, that multiplication and division by the same number are inverse operations (e.g., $3 \times \square = 12$ is the same as $12 \div 3 = \square$ and $\square = 4$).

d. Demonstrate the effect of place value when multiplying whole numbers by 10.

e. Write a story problem that relates to a given addition, subtraction, or multiplication equation, and write a number sentence to solve a problem related to the students' environment.

Objective 4: Compute and solve problems involving addition and subtraction of 3- and 4-digit numbers and basic facts of multiplication and division.

a. Use a variety of methods to facilitate computation (e.g., estimation, mental math strategies, paper and pencil).

b. Find the sum or difference of numbers, including monetary amounts, using models and strategies such as expanded form, compensation, partial sums, and the standard algorithm.

c. Compute basic multiplication facts (0-10) and related division facts using a variety of strategies based on properties of addition and multiplication (i.e., commutative, associative, identity, zero, and the distributive properties).

Mathematical Language and Symbols Students Should Use

sum, difference, expanded form, factor, product, array, multiple, numerator, denominator, halves, thirds, fourths, sixths, eighths, divisor, dividend, quotient, greater than, less than, equal to, <, >, =, standard form, word form

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Exploratory Concepts and Skills

√ Extend multiplication and division to larger-digit numbers.

√ Use concrete objects and visual models to add and subtract common decimals.

√ Investigate the distributive property of multiplication over addition for single-digit multipliers (e.g., 7×15 is equivalent to $7 \times (10 + 5)$ is equivalent to $(7 \times 10) + (7 \times 5)$.

Standard 2: Students will use patterns, symbols, operations, and properties of addition and multiplication to represent and describe simple number relationships.

Objective 1: Create, represent, and analyze growing patterns.

a. Create and extend growing patterns using objects, numbers, and tables.

b. Describe how patterns are extended using manipulatives, pictures, and numerical representations.

Objective 2: Recognize, represent, and simplify simple number relationships using symbols, operations, and properties.
    a. Represent numerical relationships as expressions, equations, and inequalities.
    b. Solve equations involving equivalent expressions (e.g., $6 + 4 = \Delta + 7$).
    c. Use the >, <, and = symbols to compare two expressions involving addition and subtraction (e.g., $4 + 6 \square 3 + 2$; $3 + 5 \square 16 - 9$).
    d. Recognize and use the commutative, associative, distributive, and identity properties of addition and multiplication, and the zero property of multiplication.

Mathematical Language and Symbols Students Should Use
growing patterns, expressions, equations, <, >, =, /,⌐, ÷, ·,×

Standard 3: Students will describe and analyze attributes of two-dimensional shapes.

Objective 1: Describe and compare attributes of two-dimensional shapes.
    a. Identify, describe, and classify polygons (e.g., pentagons, hexagons, octagons).
    b. Identify attributes for classifying triangles (e.g., two equal sides for the isosceles triangle, three equal sides for the equilateral triangle, right angle for the right triangle).
    c. Identify attributes for classifying quadrilaterals (e.g., parallel sides for the parallelogram, right angles for the rectangle, equal sides and right angles for the square).
    d. Identify right angles in geometric figures, or in appropriate objects, and determine whether other angles are obtuse (greater) or acute (less) than a right angle.

Objective 2: Demonstrate the meaning of congruence through applying transformations.
    a. Demonstrate the effect of reflection, translation, or rotation using objects.
    b. Determine whether two polygons are congruent by reflecting, translating, or rotating one polygon to physically fit on top of the other.

Objective 3: Solid Figures-including volume
    a. Describe, Identify, and create geometric shapes.
    • Identify, and draw points, lines, line segments and end points.
    • Identify and draw lines symmetry on triangles, squares, circles, and rectangles.
    • Determine whether an angle is right, obtuse, or acute by comparing the angle to the corner of a rectangle.
    • Classify polygons (e.g., quadrilaterals, pentagons, hexagons, octagons) by the number of sides and corners.
    • Identify, make, and describe cubes (e.g., a cube has 6 square faces, 8 vertices, and 12 edges).
    b. Describe spatial relationships.
    • Give directions to reach a location.
    • Use ordered pairs (A, 1) or region to locate positions on a map.
    • Demonstrate and use horizontal and vertical lines.

Standard 4: Students will select and use appropriate units and measurement tools to solve problems.

Objective 1: Identify and describe measurable attributes of objects and units of measurement.
      a. Describe the relationship among *metric* units of length (i.e., millimeter, centimeter, meter), between *metric* units of capacity (i.e., milliliter liter), and between *metric* units of weight (i.e., gram, kilometer).
      b. Identify the correct unit measure of distance and its relationship to other *customary* units of length.
      c. Describe the relationship among *customary* units of *capacity* (i.e., cup, pint, quart, and gallon).
      d. Estimate length, capacity and weight using *metric* and *customary* units.

Objective 2: Determine measurements using appropriate tools and formulas.
      a. Measure the length of objects to the nearest centimeter, meter, quarter-inch, foot, and yard.
      b. Measure *capacity* using milliliters, liters, cups, pints, quarts, and gallons and measure weight using grams, kilograms, and pounds.
      c. Read, tell, and write time to the nearest minute, identifying a.m. and p.m.
      d. Read and record the temperature to the nearest degree, in Fahrenheit, using a thermometer.
      e. Determine the value of a combination of coins and bills that total $20.00 or less.
      f. Count back change for a single-item purchase and determine the amount of change to be received from a multiple-item purchase.
      g. Determine possible perimeters, in whole units, for a rectangle with a *fixed area* and determine possible areas when given a rectangle with a *fixed perimeter*.
      i. Time elapsed in hours.

Mathematical Language and Symbols Students Should Use
measure, unit, metric system, customary system, length, pound, ounce, centimeter, meter, inch, foot, yard, capacity, weight, perimeter, volume, pint, miles, kilometers, grams and kilograms.

Exploratory Concepts and Skills
√ Determine the value of a combination of coins and bills.
√ Count back change from a single purchase.

Standard V: Students will collect and organize data to make predictions and identify basic concepts of probability.

Objective 1: Collect, organize, and display data to make predictions.
     a. Collect, read, represent, and interpret data using tables, graphs, and charts, including keys (e.g., pictographs, bar graphs, frequency tables, line plots).
     b. Make predictions based on a data display.
Objective 2: Identify basic concepts of probability.
     a. Describe the results of events using the terms "certain," "likely," "unlikely," and "impossible."
     b. Conduct simple probability experiments, record possible outcomes systematically, and display results in an organized way (e.g., chart, graph).
     c. Use results of simple probability experiments to describe the likelihood of a specific outcome in the future.

Mathematical Language and Symbols Students Should Use
data, table, chart, graph, frequency table, line plot, pictograph, bar graph, likely, certain, outcome, impossible outcome

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Exploratory Concepts and Skills
√ Predict outcomes of simple experiments.

Fourth Grade Mathematics

By the end of grade four, students develop quick recall of the basic multiplication facts and related division facts. They develop fluency with efficient procedures for multiplying multi-digit whole numbers, understand why the procedures work, and use them to solve problems. Students recognize decimal notation as an extension of the base-ten system. They relate their understanding of fractions to decimals. They generate equivalent fractions, simplify fractions, and identify equivalent fractions and decimals; compare and order whole numbers, simple fractions, and decimals to hundredths; and estimate decimal or fractional amounts in problem solving.

Students use transformations, including those that produce line and rotational symmetry. Students understand area as a measurable attribute of two-dimensional regions. They select appropriate units, strategies, and tools for solving problems that involve measuring area. They connect area measure to the area model for multiplication as a way to justify the formula for the area of a rectangle.

Curriculum Focal Points
- *Number and Operations and Algebra:* Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication.
- *Number and Operations:* Developing an understanding of decimals, including the connections between fractions and decimals.
- *Measurement:* Developing an understanding of area and determining the areas of two-dimensional shapes.
  (See Appendix I for complete document)

Standard 1: Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.

Objective 1: Demonstrate multiple ways to represent whole numbers and decimals, from hundredths to one million, and fractions.
    a. Read and write numbers in standard and expanded form.
    b. Demonstrate multiple ways to represent whole numbers and decimals by using models and symbolic representations (e.g., 36 is the same as the square of six, three dozen, or 9 x 4).
    c. Identify the place and the value of a given digit in a seven-digit numeral, including decimals to hundredths, and round to the nearest tenth.
    d. Divide regions, lengths, and sets of objects into equal parts using a variety of models and illustrations.
    e. Name and write a fraction to represent a portion of a unit whole, length, or set for halves, thirds, fourths, fifths, sixths, eighths, and tenths.
    f. Identify and represent square numbers using models and symbols.
    g. Classify whole numbers from 2 to 20 as _prime_ or _composite_ using models and symbols 2-100 and 0 and 1 as neither prime nor composite, using models.

Objective 2: Analyze relationships among whole numbers, commonly used fractions, and decimals to hundredths.
    a. Compare the relative size of numbers (e.g., 475 is comparable to 500; 475 is small compared to 10,000 but large compared to 98).
    b. Order whole numbers up to six digits, simple fractions, and decimals using a variety of methods (e.g., number line, fraction pieces) and use the symbols <, >, and = to record the relationships.
    c. Identify a number that is between two given numbers (e.g., 3.2 is between 3 and 4; find a number between 0.1 and 0.2).
    d. Identify equivalences between fractions and decimals by connecting models to symbols.
    e. Generate equivalent fractions and simplify fractions using models, pictures, and symbols.
    f. Identify the number that is 100 more, 100 less, 1000 more, and 1000 less than any whole number up to 10,000

Objective 3: Model and illustrate meanings of multiplication and division of whole numbers and the addition and subtraction of fractions.
    a. Model multiplication (e.g., equal-sized groups, rectangular arrays, area models, equal intervals on the number line), place value, and properties of operations to represent multiplication of a one- or two-digit factor by a two-digit factor and connect the representation to an algorithm.
    b. Use rectangular arrays to interpret factoring (e.g., find all rectangular arrays of 36 tiles and relate the dimensions of the arrays to factors of 36).
    c. Demonstrate the mathematical relationship between multiplication and division (e.g., 3 x □ = 12 is the same as 12 ÷ 3 = □ and □ = 4) and use that relationship to explain that division by zero is not possible.
    d. Represent division of a three-digit dividend by a one-digit divisor, including whole number remainders, using a variety of methods (e.g., rectangular arrays, manipulatives, pictures), and connect the representation to an algorithm.
    e. Use models to add and subtract simple fractions where one single-digit denominator is 1, 2, or 3 times the other (e.g., 2/4 + 1/4; 3/4 - 1/8).

Objective 4: Solve problems involving multiplication and division of whole numbers and addition and subtraction of simple fractions and decimals.

      a. Use estimation, mental math, paper and pencil, and calculators to perform mathematical calculations and identify when to use each one appropriately.

      b. Select appropriate methods to solve a single operation problem and estimate computational results or calculate them directly, depending on the context and numbers involved in a problem.

      c. Write a story problem that relates to a given multiplication or division equation, and select and write a number sentence to solve a problem related to the environment.

      d. Solve problems involving simple fractions and interpret the meaning of the solution (e.g., A pie has been divided into six pieces and one piece is already gone. How much of the whole pie is there when Mary comes in? If Mary takes two pieces, how much of the whole pie has she taken? How much of the pie is left?)

Objective 5: Compute problems involving multiplication and division of whole numbers and addition and subtraction of simple fractions and decimals.

      a. Demonstrate quick recall of basic multiplication and division facts.

      b. Multiply up to a three- digit factor by a two-digit factor with fluency, using efficient procedures.

      c. Divide up to a three-digit dividend by a one-digit divisor with fluency, using efficient procedures.

      d. Add and subtract decimals and simple fractions where one single-digit denominator is 1, 2, or 3 times the other (e.g., $2/4 + 1/4 = 3/4$; $1/3 – 1/6 = 1/6$).

Mathematical Language and Symbols Students Should Use
sum, difference, expanded form, standard form, square number, dividend, divisor, quotient, factor, product, array, multiple, numerator, denominator, sixths, eighths, tenths, equivalent, estimate, $<$, $>$, $=$, $\neq$, prime composite

Exploratory Concepts and Skills
√ Use concrete objects and visual models to add and subtract common decimals.
√ Explore numbers less than zero by extending the number line and by using familiar applications such as temperature.
√ Investigate the concept of ratio (e.g., the number of students to the number of teachers).

Standard 2: Students will use patterns and relations to represent mathematical problems and number relationships.

Objective 1: Identify, analyze, and determine rules for describing numerical patterns involving operations and non-numerical growing patterns.

      a. Analyze growing patterns using objects, pictures, numbers, and tables to determine a rule for the pattern.

      b. Recognize, represent, and extend simple patterns involving multiples and other number patterns (e.g., square numbers) using objects, pictures, numbers, and tables.

      c. Identify simple relationships in real-life contexts and use mathematical operations to describe the pattern (e.g., the number of legs on a given number of chairs may be determined by counting by fours or by multiplying the number of chairs by 4).

Objective 2: Use algebraic expressions, symbols, and properties of the operations to represent, simplify, and solve mathematical equations and inequalities.

      a. Use the order of operations to evaluate, simplify, and compare mathematical expressions involving the four operations, parentheses, and the symbols <, >, and = (e.g., 2x (4 - 1) + 3; of the two quantities 7 - (3 - 2) or (7 - 3) - 2, which is greater?).

      b. Express single-operation problem situations as equations and solve the equation.

      c. Recognize that a symbol represents the same number throughout an equation or expression (e.g., Δ + Δ = 8; thus, Δ = 4).

      d. Describe and use the commutative, associative, distributive, and identity properties of addition and multiplication, and the zero property of multiplication.

Mathematical Language and Symbols Students Should Use
growing pattern, order of operations, parentheses, inequality, expression, equation, associative property, commutative property, distributive property, zero property of multiplication, >, <, =, ×, ·, ⌐, ÷, /, -

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Exploratory Concepts and Skills
√ Use concrete materials to build an understanding of equality and inequality.
√ Explore properties of equality in number sentences (e.g., when equals are added to equals, then the sums are equal; when equals are multiplied by equals, then the products are equal).

Standard 3: Students will understand attributes and properties of plane geometric objects and spatial relationships.

Objective 1: Identify and describe attributes of two-dimensional geometric shapes.

      a. Name and describe lines that are parallel, perpendicular, and intersecting.

      b. Identify and describe right, acute, obtuse, and straight angles.

      c. Identify and describe the radius and diameter of a circle.

      d. Identify and describe figures that have line symmetry and rotational symmetry.

Objective 2: Specify locations using grids and maps.

      a. Locate coordinates in the first quadrant of a coordinate grid.

      b. Give the coordinates in the first quadrant of a coordinate grid.

      c. Locate regions on a map of Utah.

      d. Give the regions of a position on a map of Utah.

Objective 3: Visualize and identify geometric shapes after applying transformations.

      a. Identify a translation, rotation, or a reflection of a geometric shape.

      b. Recognize that 90°, 180°, 270°, and 360° are associated, respectively, with 1/4, 1/2, 3/4, and full turns.

Mathematical Language and Symbols Students Should Use
parallel, perpendicular, intersecting lines, right angle, acute angle, obtuse angle, straight angle, circle, radius, diameter, line symmetry, rotational symmetry, coordinate, first quadrant, degree, translate, rotate, reflect, transformation, reflex, angle, congruent, parallelogram, rhombus, trapezoid

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Exploratory Concepts and Skills

√ Analyze results of transformations (e.g., translations, rotations, reflections) on two dimensional shapes.
√ Investigate two-dimensional representations of three-dimensional objects.

Standard 4: Students will describe relationships among units of measure, use appropriate measurement tools, and use formulas to find area measurements.

Objective 1: Describe relationships among units of measure for length, capacity, and weight, and determine measurements of angles using appropriate tools.

> a. Describe the relative size among metric units of length (i.e., millimeter, centimeter, meter), between metric units of capacity (i.e., milliliter, liter), and between metric units of weight (i.e., gram, kilogram).
> b. Describe the relative size among customary units of capacity (i.e., cup, pint, quart, gallon).
> c. Estimate and measure capacity using milliliters, liters, cups, pints, quarts, and gallons, and measure weight using grams and kilograms.
> d. Recognize that angles are measured in degrees and develop benchmark angles (e.g., 45°, 60°, 120°) using 90° angles to estimate angle measurement.
> e. Measure angles using a protractor.
> f. a. Measure the length of objects to the nearest centimeter, meter, quarter-inch, foot, and yard.
> g. Read, tell, and write time to the nearest minute, identifying a.m. and p.m.
> h. Read and record the temperature to the nearest degree, in Fahrenheit, using a thermometer.
> i. Determine the value of a combination of coins and bills that total $20.00 or less.
> j. Count back change for a single-item purchase and determine the amount of change to be received from a multiple-item purchase.

Objective 2: Recognize and describe area as a measurable attribute of two-dimensional shapes and calculate area measurements.

> a. Quantify area by finding the total number of same-sized units of area needed to fill the region without gaps or overlaps.
> b. Recognize that a square that is 1 unit on a side is the standard unit for measuring area.
> c. Develop the area formula for a rectangle and connect it with the area model for multiplication.
> d. Develop and use the area formula for a right triangle by comparing with the formula for a rectangle (e.g., two of the same right triangles makes a rectangle).
> e. Develop, use, and justify the relationships among area formulas of triangles and parallelograms by decomposing and comparing with areas of right triangles and rectangles.
> f. Determine possible perimeters, in whole units, for a rectangle with a fixed area, and determine possible areas when given a rectangle with a fixed perimeter.

Mathematical Language and Symbols Students Should Use
millimeter, centimeter, meter, milliliter, liter, gram, kilogram, cup, pint, quart, gallon, area, perimeter, Fahrenheit, Celsius, ° (degree), range, median, mode

Exploratory Concepts and Skills

| √ Investigate perimeter of rectangles and squares. |
| √ Investigate area of trapezoids. |

Standard 5: Students will interpret and organize collected data to make predictions, answer questions, and describe basic concepts of probability.

Objective 1: Collect, organize, and display data to answer questions.
      a. Identify a question that can be answered by collecting data.
      b. Collect, read, and interpret data from tables, graphs, charts, surveys, and observations.
      c. Represent data using frequency tables, bar graphs, line plots, and stem and leaf plots.
      d. Identify and distinguish between clusters and outliers of a data set.

Objective 2: Describe and predict simple random outcomes.
      a. Describe the results of experiments involving random outcomes as simple ratios (e.g., 4 out of 9, 4/9).
      b. Conduct simple probability experiments, with and without replacement, record possible outcomes systematically, and display results in an organized way.
      c. Use the results of simple probability experiments, with and without replacement, to describe the likelihood of a specific outcome in the future.

| **Mathematical language and Symbols Students Should Use** |
| Data, line plot, line graph, bar graph, stem and leaf plot, cluster, outlier, frequency table, probability |

| **Exploratory Concepts and Skills** |
| √ Explore mean, median, mode, and range. |
| √ Explore minimum and maximum values for a set of data. |

Fifth Grade Mathematics

By the end of grade five, students increase their facility with the four basic arithmetic operations applied to whole numbers, fractions, and decimals. They locate integers on a number line and ordered pairs of integers on the coordinate plane. They determine rules for numerical patterns, work with expressions including order of operations, and solve single-operation equations involving a single variable. They classify angles, triangles, and quadrilaterals, and analyze relationships among lines, triangles and quadrilaterals. They recognize and determine surface area and volume of three-dimensional shapes, including right prisms. Students understand the concepts of mean, median, mode, and range of data sets and can calculate them. They use line plots, bar graphs, and line graphs to record and analyze data.

> **Curriculum Focal Points**
> - *Number and Operations and Algebra*: Developing an understanding of and fluency with division of whole numbers.
> - *Number and Operations*: Developing an understanding of and fluency with addition and subtraction of fractions and decimals.
> - *Geometry and Measurement and Algebra:* Describing three-dimensional shapes and analyzing their properties, including volume and surface area.
> - (See Appendix I for complete document)

Standard 1: Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 1: Represent whole numbers and decimals from thousandths to one billion, fractions, percents, and integers.
       a. Read and write numbers in standard and expanded form.
       b. Demonstrate multiple ways to represent whole numbers, decimals, fractions, percents, and integers using models and symbolic representations (e.g., 108 = 2 x 50 + 8; 108 = 102 + 8; 90% = 90 out of 100 squares on a hundred chart).
       c. Identify, read, and locate fractions, mixed numbers, decimals, and integers on the number line.
       d. Represent repeated factors using exponents.
       e. Describe situations where integers could be used in the students' environment.

Objective 2: Explain relationships and equivalencies among integers, fractions, decimals, and percents.
       a. Compare fractions by finding a common denominator.
       b. Order integers, fractions (including mixed numbers), and decimals using a variety of methods, including the number line.
       c. Rewrite mixed numbers and improper fractions from one form to the other and represent each using regions, sets of objects, or line segments.
       d. Represent commonly used fractions as decimals and percents in a variety of ways (e.g., models, fraction strips, pictures, calculators, algorithms).
       e. Model and calculate equivalent forms of a fraction (including simplest form).
       f. Rename whole numbers as fractions with different denominators (e.g., 5 = 5/1, 3 = 6/2, 1 = 7/7).

Objective 3: Use number theory concepts to develop and use divisibility tests; classify whole numbers to 50 as prime, composite, or neither; and find common multiples and factors.
       a. Identify patterns with skip counting and multiples to develop and use divisibility tests for determining whether a whole number is divisible by 2, 3, 5, 6, 9, and 10.
       b. Use strategies for classifying whole numbers to 50 as prime, composite, or neither.
       c. Rewrite a composite number between 2 and 50 as a product of only prime numbers.
       d. Find common multiples and factors and apply to adding and subtracting fractions.

Objective 4: Model and illustrate meanings of multiplication and division.
       a. Represent division-with-remainder using whole numbers, decimals, or fractions.
       b. Describe the effect of place value when multiplying and dividing whole numbers and decimals by 10, 100, and 1,000.

c. Model multiplication of fractions and decimals (e.g., tenths multiplied by tenths, a whole number multiplied by tenths, or a whole number with tenths multiplied by tenths) in a variety of ways (e.g., manipulatives, number line and area models, patterns).

Objective 5: Solve problems involving one or two operations.
a. Determine when it is appropriate to use estimation, mental math strategies, paper and pencil, and algorithms.
b. Make reasonable estimations of fraction and decimal sums, differences, and products, including knowing whether results obtained using a calculator are reasonable.
c. Write number sentences that can be used to solve a two-step problem.
d. Interpret division-with-remainder problems as they apply to the environment (e.g., If there are 53 people, how many vans are needed if each van holds 8 people?).

Objective 6: Demonstrate proficiency with multiplication and division of whole numbers and compute problems involving addition, subtraction, and multiplication of decimals and fractions.
a. Multiply multi-digit whole numbers by a three-digit whole number with fluency, using efficient procedures.
b. Divide multi-digit dividends by a two-digit divisor with fluency, using efficient procedures.
c. Add and subtract decimals with fluency, using efficient procedures.
d. Add and subtract fractions with fluency.
e. Multiply fractions.

Mathematical Language and Symbols Students Should Use
prime, composite, exponent, fractions, numerator, denominator, common denominator, common factor, common multiple, decimals, percents, divisible, divisibility, equivalent fractions, integer, dividend, quotient, divisor, factor, order of operations, simplest terms, various symbols for multiplication and division, mixed numeral, improper fraction

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Exploratory Concepts and Skills
√ Extend classification of whole numbers from 0-100 as prime, composite, or neither.
√ Apply rules of divisibility.
√ Explore adding and subtracting integers.
√ Divide multi-digit dividends by a two-digit divisor.

Standard 2: Students will use patterns and relations to represent and analyze mathematical problems and number relationships using algebraic symbols.

Objective 1: Identify, analyze and determine a rule for predicting and extending numerical patterns involving operations whole numbers, decimals, and fractions.
a. Analyze and make predictions about numeric patterns, including decimals and fractions.
b. Determine a rule for the pattern using organized lists, tables, objects, and variables.

Objective 2: Use algebraic expressions, inequalities, or equations to represent and solve simple real-world problems.
  a. Use properties and the order of operations involving addition, subtraction, multiplication, division, and the use of parentheses to compute with whole numbers, decimals, and fractions.
  b. Use patterns, models, and relationships as contexts for writing and solving simple equations and inequalities with whole number solutions (e.g., $6x = 54$; $x + 3 = 7$).

Mathematical Language and Symbols Students Should Use
variety , and * as symbols for of symbols for multiplication and division such as x, multiplication and ÷, and a fraction bar (/ or —) as division symbols; , variable, order of operations, parentheses, inequality, expression, equation, associative property, commutative property, distributive property

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Exploratory Concepts and Skills
√ Solve multi-step equations.
√ Construct and analyze tables involving equivalent ratios.

Standard 3: Students will use spatial reasoning to recognize, describe, and analyze geometric shapes and principles.

Objective 1: Describe relationships between two- and three-dimensional shapes and analyze attributes and properties of geometric shapes.
  a. Draw, label, and describe line segments, rays, lines, parallel lines, and perpendicular lines.
  b. Draw, label, and define an angle as two rays sharing a common endpoint (vertex).
  c. Classify triangles and quadrilaterals and analyze the relationships among the shapes in each classification (e.g., a square is a rectangle).
  d. Relate pyramids and right prisms to the two-dimensional shapes (nets) from which they were created.
  e. Identify properties and attributes of solids (i.e., right prisms, pyramids, cylinders, cones) and describe them by the number of edges, faces, and vertices as well as the types of faces.

Objective 2: Specify locations in a coordinate plane.
  a. Locate points defined by ordered pairs of integers.
  b. Write an ordered pair for a point in a coordinate plane with integer coordinates.
  c. Specify possible paths between locations on a coordinate plane and compare distances of the various paths.

Mathematical Language and Symbols Students Should Use
perpendicular and parallel lines, rays, angles (acute, obtuse, right, straight), triangles (equilateral, isosceles, scalene, right, acute, obtuse), vertex, vertices, edge, face, corresponding angles, similar, polygon, pyramid, right prism

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Exploratory Concepts and Skills
√ Compare corresponding angles of two triangles and determine whether the triangles are similar.
√ Rotate a shape around a fixed point and identify the location of the new vertices.

√ Translate a polygon either horizontally or vertically on a coordinate grid and identify the location of the new vertices. √ Reflect a shape across either the x- or y-axis and identify the location of the new vertices.

Standard 4: Students will determine area of polygons and surface area and volume of three-dimensional shapes.

Objective 1: Determine the area of polygons and apply to real-world problems.
      a. Determine the area of a trapezoid by the composition and decomposition of rectangles, triangles, and parallelograms.
      b. Determine the area of irregular and regular polygons by the composition and decomposition of rectangles, triangles, and parallelograms.
      c. Compare areas of polygons using different units of measure within the same measurement system (e.g., square feet, square yards).

Objective 2: Recognize, describe, and determine surface area and volume of three-dimensional shapes.
      a. Quantify volume by finding the total number of same-sized units of volume needed to fill the space without gaps or overlaps.
      b. Recognize that a cube having a 1 unit edge is the standard unit for measuring volume expressed as a cubic unit.
      c. Derive and use the formula to determine the volume of a right prism with a triangular or rectangular base.
      d. Relate the formulas for the areas of triangles, rectangles, or parallelograms to the surface area of a right prism.
      e. Derive and use the formula to determine the surface area of a right prism and express surface area in square units.

Mathematical Language and Symbols Students Should Use area, volume, surface area, volume, right prism

Exploratory Concepts and Skills √ Investigate pi as the ratio of the circumference to the diameter of a circle. √ Determine the volume of a right prism with various bases.

Standard 5:   Students will construct, analyze, and construct reasonable conclusions from data and apply basic concepts of probability.

Objective 1: Formulate and answer questions using statistical methods to compare data, and propose and justify inferences based on data.
      a. Construct, analyze, and display data using an appropriate format (e.g., line plots, bar graphs, line graphs).
      b. Recognize the differences in representing categorical and numerical data.
      c. Identify minimum and maximum values for a set of data.
      d. Identify and calculate the mean, median, mode, and range.

Objective 2: Apply basic concepts of probability.
>a. Describe the results of experiments involving random outcomes using a variety of notations (e.g., 4 out of 9, 4/9).
>b. Recognize that probability is always a value between 0 and 1 (inclusively).
>c. Express the likelihood of an outcome in a simple experiment as a value between 0 and 1 (inclusively).

Mathematical Language and Symbols Students Should Use
data, minimum values, maximum values, mean, median, mode, average, range

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Exploratory Concepts and Skills
√ Explore the differences in representing categorical and numerical data.